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Student's Competency in Solving and Creating Mathematical Problem in Pre-Service Training Program

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Abstract

Student as a mathematics teacher candidate must have competence in solving and creating mathematical problems. This paper describes how the two competencies are mastered by students. On the one competence, problem solving ability is reviewed by using four aspects: (1) conceptual and procedural understanding, (2) strategy knowledge, (3) mathematical communication ability and (4) accuracy. On the other competence, student's creativity in developing mathematical problem is evaluated based on: (1) fluency, (2) flexibility and (3) novelty. Research subjects are students who take Integral Calculus course in the academic year of 2012. Data are collected through documentation, observation and interview and then analyzed quantitatively and qualitatively. Students show their abilities and some difficulties in both solving and creating mathematical problem.

Keywords: Problem solving; creating problem; mathematical problem; pre-service training program; calculus integral

1.0 INTRODUCTION

Students have learned Calculus Integral since senior high school based on Indonesian Curriculum 2006 with three student's basic competencies; i.e. (1) understanding the concept of definite and indefinite integral; (2) computing definite and indefinite integral of simple algebra and trigonometric function; and (3) using integral to calculate area of plane region and volume of solids of revolution. In National Final Examination, the percentage of students who is able to solve volume of solid by integral in three years is as graph shown in Figure 1 below (BSNP: 2009, 2010, and 2011). Based on the graph, Indonesian students understanding of the integral application is only 72% in average and the one of Yogyakarta students is only 60% in average. Generally, senior high school teachers apply expository/traditional method in teaching and learning mathematics (Zulkardi, 2005) and it causes the student result is not satisfy yet. A lot of students learn mathematics by rotting memorization and understanding integral through procedural impractical applications. In their daily practice, teachers perform their lesson following this sequence: opening-example-exercise-closing (Sembiring, Hadi & Dolk, 2008).

Calculus Integral is one of the compulsory courses from undergraduate students of Mathematics Education Program Study (MEPS), Physics Education Program Study (PEPS), and Science Education Program Study (SEPS) in Mathematics and Science Faculty (MSF) of Yogyakarta State University (YSU). Some students who are not successful in the regular semester can take the course again in the additional semester which is conducted from July until August every year. For example, in 2012, there were 31% of 170 students of Mathematics Department who took again (twice) the Calculus Integral in the additional semester, see Figure 2.

To get more information, some students were interviewed after they followed integral calculus course. They explained that they did not understand how to use the disc method, washer method or shell method and also how to derive the formula appropriately; for example, how to find the volume if the region between $y = 1$ and $y = -x^2 + 3x + 1$ is rotated 360^o on $y = 5$ axis or $x = 4$ axis. Especially, they did not know ideas of partition and its relation with definite integral symbol.

Based on the Law Act No. 16 of 2007 of Indonesia Ministry of Education, every teacher must have four prior competencies; i.e. competency on pedagogy, character, social, and professional. Hence, students of pre-service training program, as teacher candidates, must understand concepts and principles in mathematics, are able to derive the formulas in mathematics, prove lemma and theorem, solve mathematical problem, and know how to teach mathematics. Student's ability to understand mathematics is the most important thing which is showed by students who are able to solve and create mathematical problems.

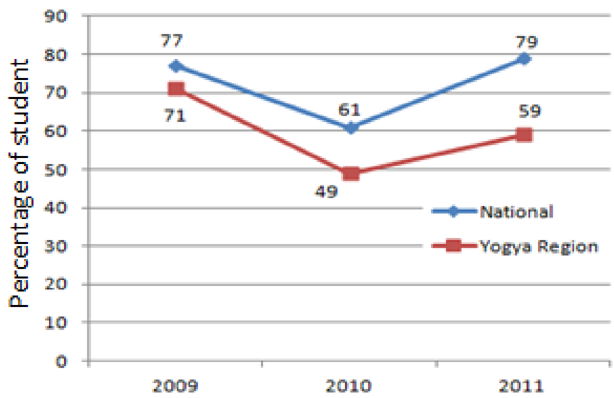


Figure 1 Percentage of student's volume-integral ability in national final examination

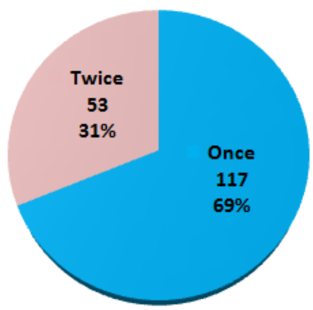


Figure 2 Percentage of student according to how much time they follow calculus integral

Lester (Branca, 1980) states that "*Problem solving is the heart of mathematics*". In line with the statement, Indonesian Curriculum 2006 has stood that problem solving ability is the core for mathematics instruction since elementary until senior high school level. Mathematics teachers must use contextual problems as a starting point in their instruction and then teach how to solve the problem by means of understanding problem, making mathematical models, solving the model, and interpreting its solution. It means that a teacher candidate must be able to solve a non-routine contextual problem in his pre-service training program. Bell (1978: 310) defines "Mathematical problem solving is the resolution of a situation in mathematics which is regarded as a problem by the person who resolves it." There are five types of mathematical problem, i.e. Recognition exercises, algorithmic exercises, application problems, open-search problems and problem situation (Buts, 1980:24). Mathematical problems in this paper are in the third, fourth and fifth type. The problems are not computational exercises which have clear strategy or puzzle having no solution conditions but between two of them (Schoen and Oehmke, 1980: 216).

In assessing student's problem solving process, NCTM (2000: 402), on the one hand, releases four standards that consist of (1) building mathematical knowledge through problem solving, (2) solving problem that arises in mathematics and in other contexts, (3) applying and adopting a variety strategies to solve problem, and (4) monitoring and reflecting the process of mathematical problem solving. On the other hand, Oregon Education Department describes five aspects of problem solving skill as follow; (1) conceptual knowledge, (2) procedural knowledge, (3) skill or strategy, (4) communication and (5) accuracy. Futhermore, Illinois State Board of Education determines three aspects; (1) mathematical knowledge, (2) strategy and (3) ability to explain. According to Sugiman and Kusumah (2010), student's problem solving can be evaluated with four components; i.e. (1) conceptual and procedural understanding, (2) strategy knowledge, (3) mathematical communication ability, and (4) accuracy. These preseding four components are used to analyze student's works in this article.

In 1950 J.P. Guilford, an American Psychologist, asked "Why is there so little apparent correlation between education and creative productiveness?" (Fasco, 2001). Consequently, as teacher candidates, students must be able to create something; it includes mathematical problems related to integral. Students must be encouraged to produce or to pose a problem and its solution. Whenever student formulate a new problem, it can foster his own knowledge and this ownership of the problem results in highly level of engagement and curiosity, as well as enthusiasm towards the process of learning mathematics (Lavy and Shriki, 2007: 130). Sternberg and Lubart propose several personality attributes that have been shown to be traits of persons considered to be creative: (a) tolerance for ambiguity, (b) willingness to surmount obstacles and persevere, (c) willingness to grow, (d) willingness to take risks, and (e) courage of one's convictions and belief in oneself (Fasco, 2001).

There are three types of problem that can be posed by students, i.e. free problem posing, semi-structured problem posing, and structured problem posing (Abu and Sayed, 2000: 59-61). Some research conducted by Tuli in 1980, Haylock in 1997, Jenses in 1973 and Kim in 2003 have applied the concepts of novelty/originality, fluency and flexibility to the concept of creativity in mathematics (Mann, 2005: 36). This paper adopts the aspect of creating problem that consists of novelty, fluency and flexibility.

■2.0 MATERIALS AND METHODS

A selected topic from Calculus Integral in this research is volume of solids because of some reasons: variety of involved mathematical concepts, multi daily life rich context, many strategies, and very essential topics. The variety of mathematical concepts is concept of area, volume, series, limit, partition, Riemann sum and indefinite integral. The appropriate daily-life contexts are pyramids building, prism monument, bucket, frying pan, glass, soccer ball, spherical balloon, and donate-shaped balloons. The multi strategies are by using formula, making partition, using Riemann sum and applying integral.

A teaching approach implemented in this research is Realistic Mathematics Education (RME). Sembiring, Hadi and Dolk (2008) and Anh (2006) apply three basic tenets of the RME in their research, namely guided reinvention, didactical phenomenology and mediating models principle. In guided reinvention principle, mathematics is not as a ready-made knowledge transferred by teachers but student should construct all concepts, ideas and principles of mathematics using his own manner and strategy. In didactical phenomenology principle, meaningful context is taken as a starting-point of mathematics teaching then it encourages students to make generalization and provide a basis for connecting among contexts, models, mathematical concepts/ideas and solutions. Doing mathematics is a student's essential activity guided by his teacher or his peers in which student can understand mathematics through his own model-of situation and model-for mathematics. In mediating models principle, a student constructs his own model that plays in changing from informal mathematics to formal mathematics. The informal models are developed by student and then gradually move to formal mathematical symbols.

Based on RME tenets, to encourage student's knowledge, we use contextual problem as a starting point and then explore its volume through four steps; i.e. using volume formula, making partition and using Excel MS, implementing Riemann sum, and applying definite integral formula, see Figure 3.

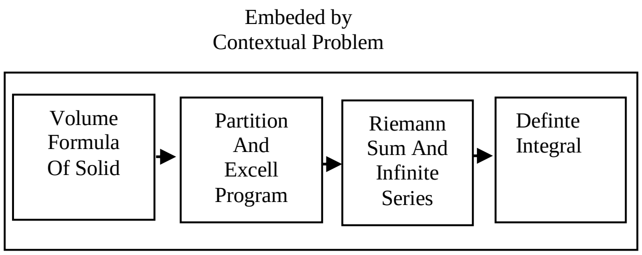


Figure 3 Four steps in finding volume of solid

The problem posed to students is arranged in a specific learning trajectory as in Figure 4. Number 1, 2, 3 and 4 meaning that students compute volume of pyramids using its formula, partition and Excel program, Riemann sum, and definite integral respectively. In the second, third, fourth and fifth column from the left, the contexts are a bucket, a frying pan, a spherical balloon and a donut shaped balloon respectively. According to the RME tenets, student may use informal mathematics whenever solving these problems then they develop his concept of definite integral in formal mathematics level. Through the three riel contexts, a student elaborates some strategies to compute the volume of sphere and of donut using definite integral.

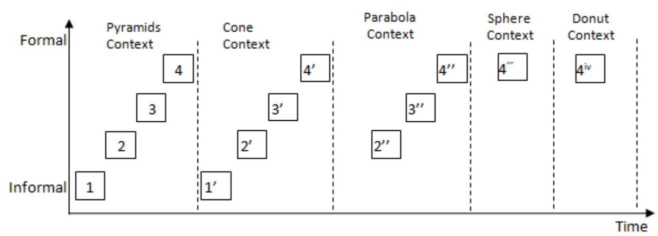


Figure 4 Learning trajectory in learning of solid volume

Teaching material is developed with learning trajectory framework like Figure 4 that refers to Calculus textbook (Larson, Hoteller and Edward, 2008). Problems of pyramids and cone volume are done by groups of students in classroom using RME approach. In addition, parabola, sphere and donut volume are explored by every group in their home. These contexts are local contexts which can be found in Yogyakarta region. Meanwhile, in the teaching learning process, students' activities and their thinking are observed and documented and then some students are interviewed to know their ideas clearly and deeply. Students' works are also collected to complete the preceding data.

3.0 RESULT AND DISCUSSION

In this session, there will be reports on the student's ability in solving and creating contextual problems respectively related to volume of solid and definite integral. Student's ability in solving problem will be reviewed from four aspects, namely conceptual and procedural understanding, strategy knowledge, mathematical communication ability and accuracy. Moreover, student's ability in creating problem will be reviewed from three aspects, namely fluency, flexibility and novelty.

Although every student has learned how to use definite integral to find volume of solid in senior high school, almost all of them still do not understand how to construct integrand function rationally and correctly. But, almost all students can integrate the polynomial function, such as linear, quadratic and cubic functions fast and adequately. The weaknesses and strengthens of student capability are valuable input for Calculus Integral course in pre-service training program.

Almost all students know about the concept of definite integral and the concept of volume but they get difficulties in making connection between the two concepts, especially in deriving the integrand function. In this situation, the teacher gives guidance in making partition of related situation, that is horizontal partition or vertical partition. Then students try to use volume formula of thin prism, thin disc or thin cylindrical shells in order to get the volume function respect to independent variable (x or y). In classroom, some slow learner students still cannot derive the volume function even though other smarter students have done that completely.

There are some procedures in applying definite integral to compute volume. First step, students remember "known pre-calculus formula", such as volume of dick ($V = \pi R^2 t$ where R = radius and t = thickness), volume of washer ($V = \pi(R^2 - r^2)t$ where R = outer radius and r = inner radius), volume of shell ($V = 2\pi pht$ where p = average radius and h = height), volume of thin prism ($V = At$ where A = base area). Second step, students write a small volume as a function respect to x or y variable, such as $\Delta V = \pi f^2(x)\,\Delta x$, $\Delta V = \pi (f^2(x) - g^2(x))\,\Delta x$, $\Delta V = 2\pi x f(x)\,\Delta x$ and $\Delta V = 2\pi x (f(x) - g(x))\,\Delta x$. Third step, students construct volume of solid with new integration formula, such as $V = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \Delta V = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \pi f^2(x)\Delta x = \int_a^b \pi f^2(x)dx$. Like Number 2 and 3 of Figure 4, student compute $\sum_{i=1}^{n} \Delta V$ for some n with Excel program and solve $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \pi f^2(x)\Delta x$ with series formula and limit. Fourth step, students integrate the function and substitute the lower and upper limit. Final step, students interpret the result back to the original problem. Not all problems are solved by students through five steps above. After students understand the meaning of definite integral then they derive general formula of solid of revolution formed by revolving some region. Then the general formula can be applied to relevant next problems.

Strategy is an essential aspect whenever students want to solve a problem. Ani, a student, wants to find the volume of frying pan and she chooses a strategy as follow, see Figure 5. She takes a photo of the frying pan and tries to find mathematics equation that match to the pan. She supposes that the equation form is parabolic equation that is $y = ax^2 + bx + c$ and then she determines four points on parabolic curve, namely (0, 0), (9, 3), (12, 6) and (14.3, 8.9). She uses substitution and elimination method to find value of a, b, and c and gets equation $y = x^2/18 - x/6$. After that she graphs the curve and chooses shell method to find its volume. Then, she derives formula of definite integral $V = 2\pi \int_0^{1,43} x\left(8,9 - \left(\frac{1}{18}x^2 - \frac{1}{6}x\right)\right)dx$ and finally she know that the volume equals 333.49 cm^3.

Budi computes the volume of the frying pan using different method, namely disc methods and Excel program. He divides interval [0, 8.9] of y axis become 50 partitions and uses formula $V \approx \sum_{n=1}^{50} \pi x^2 \Delta y$ where Δy = 8.9/50 = 0.178 and he finds that the frying pan volume equals 3085.499. Because Ani's result differs from Budi's result, Budi computes again using shell methods. He divides interval [0, 14.3] become 40 subintervals, applies formula $V \approx \sum_{n=1}^{50} 2\pi xy\Delta x$ and gets 3084.402 cm^3. It indicates that the pan problem can be solved with more than one strategy.

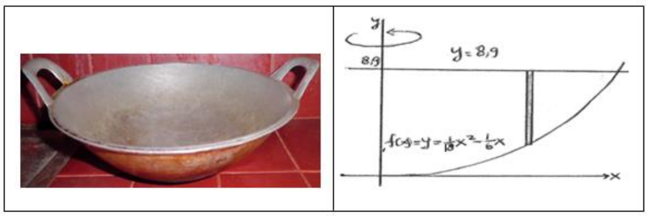


Figure 5 Ani's strategy to compute volume of frying pan

Students of pre-service training program must be able to communicate their notions clearly. In Figure 5, the student shows her capability in writing. She uses multiple representatives clearly and effectively, namely mathematical symbol of linear equation, mathematical symbol of quadratics function, mathematical symbol of rotation, axis of revolution, x and y axis, graph of linear curve, graph of parabolic curve and one area of small partition. But the student forgets to write relevant information such as point 14.3 in x axis, length of rectangular partition ($h = y_1 - y_2$), width of rectangular partition (Δx) and radius of circular shell (x). Students' orally communication competency can be seen when they discuss in their group and explain their ideas in front of the class. Most of students are able to explain and argue their opinion.

Based on explanations of Ani's result versus Budi's result in preceding paragraph, we know that there are three different students' answers. It means that sometimes students work inaccuracy. After students' works are analyzed, it is found Ani gives wrong answer. She writes that $x\left(8,9 - \left(\frac{1}{18}x^2 - \frac{1}{6}x\right)\right) = 8.9x - \frac{1}{18}x^3 - \frac{1}{6}x^2$ and it indicates that she fails in using distributive property. Actually, she understands the distributive property but, because at that time she considers to the definite integral symbols, she writes careless. Moreover, she doesn't look back at the end in order to check her answer by herself. She should read again her work, solve the problem with another strategy or use her sense of volume.

As mathematics teacher candidates, student's creativity in posing problem must be encouraged. Students do not only choose a day life context related to volume solid and definite integral but they also make its alternative solutions. Fasco (2001: 318) argues that students will be more motivated when they choose their own tasks. Many times, students still get difficulties to control their work by themselves; teacher should monitors and manages student's activities. In posing problem, students tend to choose an easy problem and try to avoid difficulties. In this situation, teacher must change students' problem such that the problem move to the Zone of Proximal Development (ZPD).

Fluently in using mathematical concepts and principles, making connection, planning strategy, executing plan and applying self reflection must be mastered by students. Planning strategy is the hardest activity for almost all students. They used to apply formula of solid volume in regular problem and with less understanding because they used to learn mathematics as a ready-made formula. As a result, students are not fluent when they meet new problems. For example, student have already know how to derive definite integral with disc and shell methods in frying pan context but when they are asked to compute volume of soccer ball, they do not know how to find upper limit and lower limit function. Generally, many students need a lot of time in solving a new contextual problem and they still need guidances from their teacher or peers.

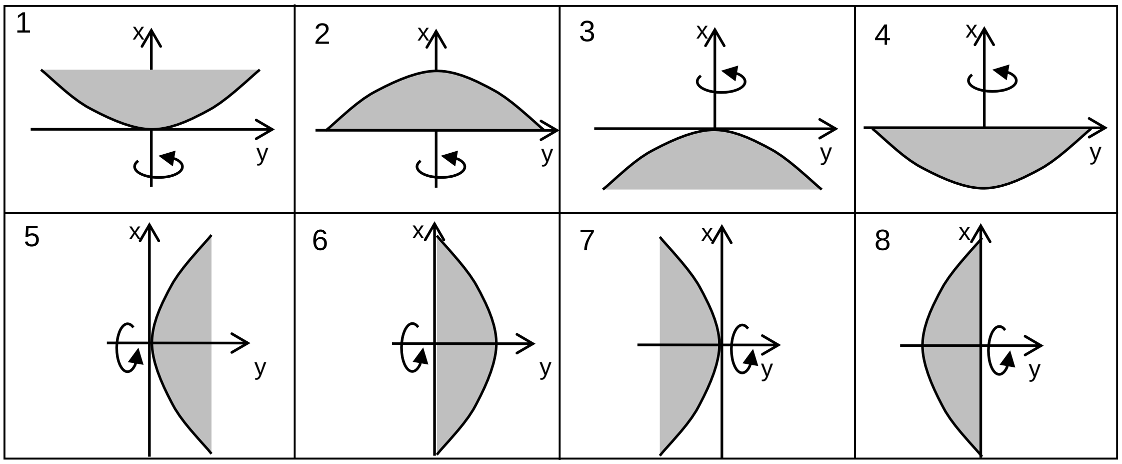
Flexibility is one aspect of creativity. Whenever students solve frying pan problem, they feel satisfy with one strategy only. Almost all students choose position of the pan as Number 1 of Picture 6 even though there are at least 8 possibility positions between pan and Cartesian Coordinates. The students' strategy is caused by the original position of the frying pan, see left photo in Picture 5. But, some of them use partition strategy with Excel program and the others use disc methods or shell methods. In order to increase students' flexibility, teachers ask them to think the other position and make alternative solution with more than one strategy.

Donate shape is the most difficult problem among five contexts in Figure 4. In the case; students may use their flexibility whenever (1) they determine method to compute volume of donate such as Riemann sum with Excel method, washer method or shell method, (2) they determine the limits function of area partition such as linier function and square root function or a couple of square root functions, (3) they determine the independent variables of integrals that is x variable or y variable and (4) they integrate the square root function.

Students show originality along teaching learning process. Originality elements that appear are (1) contexts that they choose such as glass, cup ceramic, soccer ball and cone shaped hat, (2) curves that they suppose such as linear curve, parabolic curve and circle curve, (3) step that they do whenever they use Excel program, (4) definite integral formula that they derive from original context and (5) integral techniques that they use such as substitution technique, trigonometry integration technique, trigonometry substitution technique and integration by table technique.

■4.0 CONCLUSION

Problem solving and creativity ability are essential competencies for teachers and teacher candidates. The two competencies can be fostered by implementing Realistic Mathematics Education. In general, students show their competencies in solving problems; they understand concepts of definite integral, understand procedure in using definite integral to compute volume of solid, communicate mathematically and work accurately. In addition, students also show their creativity that consists of fluency, flexibility and novelty aspect.



Picture 6 Eight possibility positions between frying pan and cartesian coordinates

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References

Abu, R. And Sayed, E. 2000. Effectiveness of Problem Posing Strategies on Perspective Mathematics Teacher's Problem Solving Performance. *Journal of Science and Mathematics Education in S.E. Asia*. 25(1): 56–69

Anh, L. T. 2006. *Applying Realistic Mathematics Education in Vietnam: Teaching Middle School Geometry*. Disertation of Postdam University.

Bell, F. H. 1978. *Teaching and Learning Mathematics (in Secondary Schools*. Second Printing. Dubuque, Iowa: Wm. C. Brown. Company.

Branca, N. A. 1980. Problem Solving as a Goal, Process and Basic Skill. *Problem Solving in School Mathematics*. Editor: Krulik, S. and Reys, R.E. Reston: National Council of Teachers of Mathematics.

Buts, T. 1980. Posing Problem Properly. *Problem Solving in School Mathematics*. Editor: Krulik, S. and Reys, R.E. Reston: National Council of Teachers of Mathematics.

Depdiknas. 2006. Indonesian Curriculum 2006: Curriculum-based Competency. Jakarta: Depdiknas.

Fasko, Daniel Jr. 2001. Education and Creativity. *Creativity Research Journal*. Lawrence Erlbaum Associates, Inc. 13(3&4): 317–327.

Larson,R., Hoteller, R., and Edward, B. H. 2008. *Essential Calculus: Early Transcendental Functions.* Boston: Houghton Mifflin Company.

Lavy, I. and Shriki, A. 2007. *Problem Posing as a Model Means for Developing Mathematical Knowledge of Prospective Teacher*. PME-31. Emek Yezreel Academic College.

Mann, E. L. 2005. Mathematical Creativity and School Mathematics: Indicator of Mathematical Creativity in Middle School Students. Dissertation. University of Connecticut.

NCTM. 2000. *Process Standard for Mathematics.* Downloaded in 2 May 2008 at http:// fcit.usf.edu/math/ resource/mathpower/standard.html.

Schoen, H. L. and Oehmke, T. 1980. A New Approach to the Measurement of Problem-solving Skills. In *Problem Solving in School Mathematics*. Editors: Krulik, S. and Reys, R. E. Reston: National Council of Teachers of Mathematics.

Sembiring, R. K., Hadi, S. And Dolk, M. 2008. Reforming mathematics learning in Indonesian classrooms through RME. *ZDM Mathematics Education*. 40:927–939.

Sugiman and Kusumah, Y.S. 2010. Dampak Pendidikan Matematika Realistik Indonesia terhadap Peningkatan Kemampuan Pemecahan Masalah Siswa SMP. IndoMS Journal on Mathematics Education (IndoMS JME). 1(1):41–52.

Zulkardi. 2005. *Peningkatan Mutu Pendidikan Matematika melalui Mutu Pembelajaran*. Downloaded in 11 July 2005 at http://www. pikiran-rakyat.com.