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The Emphasis on Generalization Strategies in Teaching Integral: Calculus Lesson Plans

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Abstract

Although integral is one of the important concepts in mathematics, most students have problems learning it. A well-designed lesson plan consisting generalization and its related activities can play an important role to overcome the students' difficulties in the learning process. In this study, generalization means going from particular to something general by looking at common things and expanding their domains of validity. The main goal of this study is to investigate the rate of using generalization by mathematics instructors in teaching of integral concepts based on lesson plans. Thus, the instructors were interviewed on what generalization means to them and to find the reasons of emphasis on generalization or lack of it in their lesson plans. Findings revealed that most instructors did not emphasise the teaching of generalization in their lesson plans. Although most of the instructors expressed that generalization can be used to extend the theorems, the interview clarified that majority of the instructors did not know about the important applications of generalization in the teaching of integral. The results demonstrated that instructors have oppressed generalization in the integral teaching process.

Keywords: Generalization; Lesson Plans; Undergraduate; Integral

Abstrak

Meskipun pengkamiran adalah salah satu konsep penting dalam matematik, kebanyakan pelajar menghadapi masalah untuk mempelajarinya. Pelan pengajaran yang direka dengan baik terdiri daripada generalisasi dan aktiviti yang berkaitan dapat memainkan peranan penting untuk mengatasi kesulitan pelajar dalam proses pembelajaran. Dalam kajian ini, generalisasi bermaksud beralih dari yang tertentu ke sesuatu yang umum dengan melihat perkara biasa dan memperluas domain kesahihannya. Matlamat utama kajian ini adalah untuk mengkaji kadar penggunaan generalisasi oleh pengajar matematik dalam mengajar konsep pengkamiran berdasarkan rancangan pelajaran. Oleh itu, para pengajar telah ditemubual tentang apa yang dimaksudkan dengan generalisasi terhadap mereka dan untuk mencari punca tentang kepentingan generalisasi atau kekurangannya dalam rancangan pelajaran mereka. Dapatan menunjukkan bahawa kebanyakan pengajar tidak menekankan pengajaran generalisasi dalam rancangan pelajaran mereka. Walaupun kebanyakan pengajar menyatakan bahawa generalisasi boleh digunakan untuk memperluaskan teorem, temuramah menjelaskan bahawa majoriti pengajar tidak mengetahui tentang aplikasi generalisasi penting dalam pengajaran pengkamiran. Keputusan menunjukkan bahawa para pengajar telah mengabaikan generalisasi dalam proses pengajaran pengkamiran.

Kata kunci: Generalisasi; Pelan Pelajaran; Sarjana Muda; Pengkamiran

1.0 INTRODUCTION

Integral is one of the most important concepts of calculus. It is a prerequisite concept for other mathematics concepts and even for other subjects (Brijlall & Ndlaz, 2019; Saepuzaman et *al*, 2017; Jones, 2013). However, most undergraduate students find that integral is the most difficult concept to learn (Moru & Qhobela, 2019; Hashemi et *al*, 2019a; Wagner, 2018; Jones et *al*, 2017; Nursyahidah & Albab, 2017; Li et *al*, 2017; Sealey, 2014; Tarmizi, 2010; Metaxas, 2007).

Difficulties to learn integral are revealed in two views; lack of conceptual understanding and weakness of problem-solving skills. According to researchers such as Metaxas (2007) and Tall (1993) conceptual understanding does not happen sufficiently and completely in the learning of integral because instructors and students focus on the symbolic aspect rather than graphical (Brijlall & Ndlaz, 2019; Moru & Qhobela, 2019; Wagner, 2018; Li et al, 2017; Tatar & Zengin, 2016), Moreover, there is no connection and relationship between graphical aspect and symbolical aspect through the learning of integral. Hence, students have difficulties in solving problems in integral because they do not use a framework or plan to solve the problems (Brijlall & Ndlaz, 2019; Nursyahidah & Albab, 2017; Li et al, 2017). In addition, they also have difficulties in using their previous knowledge and information in new areas (Villers & Garner, 2008; Mason, 2010; Tarmizi, 2010). However, generalization by making logical relationship and connection between and within concepts can overcome students' difficulties to learn integral (Hashemi et *al*, 2019b; Dorko & Weber, 2014; Stacey, 2006; Mason et *al*, 2010; Roselainy, 2009; Mitchelmore, 2002).

Generalization strategies have the potential to make logical relationships between graphical aspect and symbolical aspect of integral (Tall, 2004, 2008; Hashemi et *al*, 2013a). Furthermore, generalization can be used to establish focus on embodied aspect of presenting and reading of integral (Hashemi et *al*, 2013b; Tall, 2002). Generalization as the main process has an important role to help students in using problem solving framework to solve integral problems (Hashemi et al, 2019b; Roselainy, 2009). Thus, the use of generalization strategies in the planning of teaching and learning activities can be helpful to overcome difficulties in the learning of integral (Hashemi et al, 2019b; Dorko & Weber, 2014; Hashemi et al, 2013a, 2013b; Tall, 2008; Hashemi, 2008; Mitchelmore, 2002; Karamzadeh, 2000).

Instructional planning plays an effective role in the teaching process among mathematics instructors (West & Staub, 2003). Therefore, it is necessary for mathematics instructors to be able to design a systematic plan for teaching. Lesson plans should contain important components such as goals of lesson, relationship between the goals of study and the goals of lesson plan, evaluation of entire lesson, lesson presentation and assignments to solve in the next session (Ding & Carlson, 2013; Zazkis et *al*, 2009); Fernandez et *al*. 2003). Lesson planning is a complex cognitive skill (Wild, 2000). Thus, effective lesson planning begins with specific standards that students need to learn, and it is tied to increase students' achievement (Sri Mentari, 2018; Rettig et *al*, 2003). The main goal of this research is to investigate how much emphasis on generalization strategies was given by mathematics instructors in their lesson plans of integral. In addition, the reasons of using generalization or lack of using it in their lesson plans are discussed.

■2.0 THEORETICAL FRAMEWORK

Generalization in mathematics means looking for patterns, relationship and making connection at different levels of mathematical thinking (Tall, 2002; Sriraman, 2004; Mason et *al*, 2010). Generalization strategies are used in mathematics to show processes in broader contexts and to help problem solvers to identify the product of those processes (Tall, 2002). Tall (2002) believe that generalization can appear in three types namely; expansive, reconstructive and disjunctive.

Expansive generalization involves extending the existing information of learners without any changes in their previous ideas. Thus, new information in the expansive generalization should be close to the current information in the same area (Harel & Tall, 1991). Expansive generalization is generating without changing previous knowledge construction, for exaple; generating properties of R^2 to R^3 (Tall, 2002). Furthermore, when a person extends a concept with a change in his previous ideas or knowledge and tries to handle the new one and current, he reconstructs his cognitive area of knowledge. This kind of change is named reconstructive generalization (Tall, 2002). Reconstructive generalization is generating by changing construction of previous knowledge (Tall, 2002, 2008). For example, generating algebraic aspects of a aconcept to geometrical aspects. Then, disjunctive generalization can be used to solve and analyze problems in the higher class of knowledge. Students can use this kind of problem generation to solve new problems by adding numbers of disconnected pieces of information (Tall, 2002). For instance, using partial integral in wrong ways. Figure 1 represents the components of generalization according to Tall.

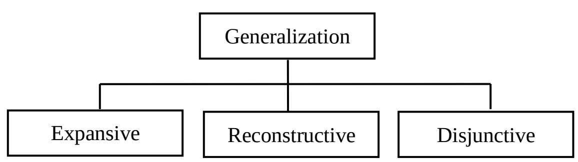

Figure1 Components of generalization

When someone wants to use generalization strategies in a teaching process, these three components should be considered (Tall, 2002). Thus, the rate of application of generalization strategies can be established by investigating the quality of utilization of these components.

■3.0 METHOD

This descriptive phenomenological study tries to highlight how much mathematics instructors use generalization strategies in their teaching of integral concepts at undergraduate level. Therefore,instructors' lesson plans on integral concepts were being investigated.

The samples of this study were seventy two calculus instructors at undergraduate level who teach mathematics in the universities of Shiraz city in Iran which it is common practice for instructors to prepare complete lesson plans for their classes. No specific criteria were used in the selection of the instructors other than the fact that they are qualified and have the experiences of teaching mathematics to the undergraduates. Around 100 instructors who taught integral were asked to submit their lesson plans and only 72 lesson plans were received. Thesamples of this study were selected because of their availability to the researchers. Their lesson plans on teaching integral concept were collected to see the rate of using generalization strategies in the process.

Instructors' lesson plans were the main instrument for data collection in this research because they can show how instructors plan to teach integral. It should be asserted tha seeing instuctores' teaching activities were not happen because it needed the time more than three monts. the The lesson plans gave information on what kind of examples used, and how the concept was presented by making connection with other notions.Hence, the lesson plans were interpreted and analyzed to see the quality of generalization strategies used. Since the generalization strategies have to imitate the three components from Tall's perspective, there was a need to have a rubric to measure and evaluate the rate of using these components in lesson plans.

The researchers designed a rubric containing a check list (CL) to measure the rate of utilization of generalization strategies in the lesson plans. The CL consists of 10 items which 2 items are for disjunctive generalization, 4 items for expansive and 4 items for reconstructive generalization. In determining the validity of rubric, two expert's opinions were sought. Table 1 presents the CL items.

Table 1 Generalization components and check list items

Components	Check List Items	
	Code	**Descriptions**
Disjunctive	**CL_1**	Introduction of integral by using connected information
	CL_2	Teaching integral by using familiar context
Expansive	**CL_3**	Providing experiences to lead meaningful understanding
	CL_4	Using more related examples to teach each part of integral
	CL_5	Moving from related examples to more general case
	CL_6	Checking solutions or reviewing related concepts
Reconstructive	**CL_7**	Generating ideas in both embodied and symbolic aspects
	CL_8	Extending the solution idea to higher level problems
	CL_9	Changing existing schema in order to widen its applicability range
	CL_{10}	Abstraction of concepts

The 10 items of CL cover the components of generalization that described earlier. The disjunctive component covers only two items because this component is a primitive generalization in low level of mathematical thinking (Harel & Tall, 1991). The expansive component which is in higher level than disjunctive involves four items of CL. The reconstructive as the highest level of generalization contains four items which are related to the reconstruction property of generalization.

In order to know the rate of using generalization strategies in the lesson plans, the CL was used to fill in data collected from instructors' lesson plans. The quality of completed CL was based on Table 2. Referring to the table, if an instructor considered an item to be sufficient, the researchers gave him/her 2 (two) for that item. If the consideration was not enough, the given score was 1 (one). Zero (0) score was given to an item in CL if there was no attention to that item. Table 2 presents the quality of scores for CL items based on the instructors' lesson plans.

Table 2 Scores for CL items

Scores	Descriptions
0	There is no emphasis of this item in the lesson plan
1	There is an emphasis of this item, but it is not enough
2	There is a sufficient emphasis of this item in the lesson plan

Thus, the limitation of using CL was possible differences between instructors' aims and the judgment of researchers about their lesson plans. For instance, if an instructor used a component or item, but (s)he did not explain it clearly in the lesson plan, the researchers would imagine that there was no consideration for that component or item. In addition, there was a limitation in using scores to items of CL in Table 2. It was difficult for the researcher to determine the exact scores if the items fell between 1 and 2. However, the collected data from CL were analyzed based on the given scores.

According to the scores, the data were numeric and were analyzed using descriptive statistical methods. The analysis was done for each item by calculating the mean and the percentage of instructors who used that item. Furthermore, descriptive analysis was also used to know the rate of using each generalization component in the instructors' lesson plans. It means that the mean of given scores and percentage of instructors were calculated for each generalization component by considering the score of the items which was related to that component. Subsequently, the percentage of utilization of each component was calculated by considering all related items of that component which was used at least once.

After analyzing the instructors' lesson plans, 12 instructors were selected randomly to be interviewed for supporting the results of this study. The instructors were asked to provide the meaning of generalization and its applications in the integral teaching process. Some interview questions are as follows; 1) what is the meaning of generalization? 2) Please give an example of generalization in integral. 3) In your opinion, how generalization can be used in teaching and learning of integral? 4) Have you ever used generalization in your lesson plan specifically in the integral concept? Please provide the rationale for your answer. 5) In your opinion, can generalization be considered as a part of lesson plan for integral concept? Please explain. Data from the interview sessions were analyzed qualitatively based on Miles & Huberman (1994) method of analysis. Three stages of this method that were used to analyze of instructor responses to the interview questions are: data reduction, data display, and conclusion drawing.

■4.0 RESULTS

Findings have been presented and explained based on quantitative results and interview results. In following, the results have been asserted in figures and tables.

Quantitative Results

The results of analysing each component were calculated based on percentage of instructors who had considered the items of related component and the mean calculated for each item in the checklist.Interpretation was included in the results of items for each component. The results of items in the CL are presented in Table 3.

Table 3 The results of analysis

	Score	CL_1	CL_2	CL_3	CL_4	CL_5	CL_6	CL_7	CL_8	CL_9	CL_{10}
Frequency of Scores among Instructors	**0**	46	49	57	60	62	64	58	61	65	63
	1	21	18	14	09	9	6	11	8	5	9
	2	5	5	1	3	1	2	3	3	2	0
Mean of score for CL		0.43	0.38	0.22	0.20	0.15	0.13	0.23	0.19	0.12	0.12
Percentage of Instructors who Used this item (Score 1 or 2)		36.1	31.9	20.0	16.6	13.8	11.1	19.4	15.2	09.7	12.5

The percentages and mean of given score indicated the weakness of using generalization strategies in lesson plans. Percentages for all items are less than 40%, and also the means are smaller than 0.5 (from 0 to 2). The data show a lack of using generalization among instructors. The results and interpretation of them in the table above prove that mathematics instructors at undergraduate level were seriously nonfeasance to use generalization in their integral concepts teaching process. The inadequacy existed in all of the components in the instructors' lesson plans. The data indicate that the instructors did not pay attention to any component of generalization namely; expansive, reconstructive and disjunctive.

The results of items which are related to disjunctive component are presented in the table below. In addition, the quality of results and interpretation of them are also being discussed at length.

Table 4 Result of disjunctive component and its related items

Descriptive Results	Score	CL_1	CL_2	Disjunctive
Frequency	**0**	46	49	95
	1	21	18	39
	2	5	5	10
Mean		0.43	0.38	0.405
Percentage of Instructors who used this item (Score 1 or 2)		36.1	31.9	43

Although using information of other concepts of calculus such as function, limitation and derivation is necessary to teach integral (Orton, 1983), the results show that only 36.1% of instructors (26 of 72) made logical connection with other concepts in introducing integral. The lowest type of generalization is to use familiar context in the teaching of calculus (Tall, 2002), but only 31.9% of the instructors (23 from 72) used familiar context to teach integral.

The results of Table 4 show that the consideration of disjunctive generalization was very weak in this study. The mean of 0.405 indicates that there was no valuable attention to disjunctive generalization among the mathematics instructors. In addition, only 31 instructors paid attention to disjunctive component in their lesson plans. According to the results in Tables 3 and 4, disjunctive component of generalization was used more than the other component in instructors' lesson plan (about 43%). Although Harel & Tall (1991) and Tall (2002) emphasize that this component is less useful than others for students and it is the lowest level of generalization, the table shows that the results are disappointing. The data indicate that instructors did not pay attention to the role of this component. Therefore, the rate of application in the teaching processes in this research was very catastrophically low.

Next, the results of expansive component and its items are presented in Table 5. This component contains four items namely; CL_3, CL_4, CL_5 and CL_6. More details are discussed by using descriptive statistical methods.

Table 5 Results of expansive generalization

Descriptive Results	Score	CL_3	CL_4	CL_5	CL_6	Expansive
Frequency	0	57	60	62	64	243
	1	14	9	9	6	38
	2	1	3	1	2	7
Mean		0.22	0.20	0.15	0.13	0.17
Percentage of Instructors who used this item (Score 1 or 2)		20	16.6	13.8	11.1	27.7

The results revealed that twenty percent of instructors (15 from 72) provided helpful experiences to lead students for meaningful understanding of integral. The mean of given scores to this item (CL_3) in Table 5 verifies the insignificant attention to this item. Furthermore, 16.6 percent of the instructors (12 of 72) prepared suitable related examples in the teaching of integral. However, the mean of given scores which related to this item (CL_4) is 0.20. It shows that less attention was given to the variety of suitable examples in the teaching of integral among the instructors.

In addition to using less related examples, 86.2% of instructors could not move from examples to a general case (CL_5). The average of given scores for this item is rated low and equal to 0.15. Thus, there was a weak consideration to move from examples to a general case. Furthermore, the mean 0.13 highlights that there was no acceptable activities for checking solution or reviewing the concepts through teaching integral (CL_6). In other words, only 11.1 percent of instructors (8 of 72) checked the solution or reviewing the concepts.

According to the results it can be claimed that there was no acceptable consideration to the expansive component of generalization among the instructors. The mean of 0.17 confirms this notion and the number of instructors who emphasized expansive component was only 20 (about 27.7%) which were less than half of all the samples.

The results of reconstructive generalization are presented in Table 6 and the discussion on the rate of utilization of this component in the instructors' lesson plan is written in the subsequent paragraphs.

Table 6. Results for reconstructive generalization

Descriptive Results	Score	CL_7	CL_8	CL_9	CL_{10}	Reconstructive
Frequency	0	58	61	65	63	247
	1	11	8	5	9	33
	2	3	3	2	0	8
Mean		0.23	0.19	0.12	0.12	0.16
Percentage of Instructors who used this item (Score 1 or 2)		19.4	15.2	09.7	12.5	22.6

It is necessary to use figures and curves to teach mathematics lessons especially in dealing with main concepts such as limitation, derivation and integral (Tall, 2008). Analysis of the data shows that only 19.4% of the instructors emphasized to generate ideas in both embodied and symbolic aspect of integral representation (12 persons of 72). The mean of given scores to the instructors' lesson plan for this item (CL_7) is 0.23. This information indicates that majority of the instructors did not use embodied and symbolic aspects to teach integral. Furthermore, only 11 instructors (15.2%) displayed their attempts to extend the solution ideas to higher level problems through the teaching integral concept. Other descriptive results such as the mean of 0.19 portrays the weakness of instructors' in extending ideas to higher level problem of integral (CL_8).

Only 7 instructors (9.7% with mean= 0.12) endeavored to change the existing schema to show the wide application of integral (CL_9) in their lesson plans. In addition, about 12.5% of instructors (9 instructors) tried to introduce integral by abstracting its properties (CL_{10}). Based on Table 6, the 9 instructors had an insufficient emphasis on abstracting the concept. Subsequently, the mean of 0.12 demonstrates that the instructors' had low intention to use reconstructive generalization.

The results in Table 6 prove that the mathematics instructors did not apply reconstructive generalization in their lesson plans. On other hand, the mean of 0.16 (22.6%) of the instructors used this component in their lesson plans. Therefore, there was lack of serious emphasis with regard to reconstructive generalization among the mathematics instructors in this study. Figure 2 presents the rate of utilization of all the generalization components and their related items based on the lesson plans.

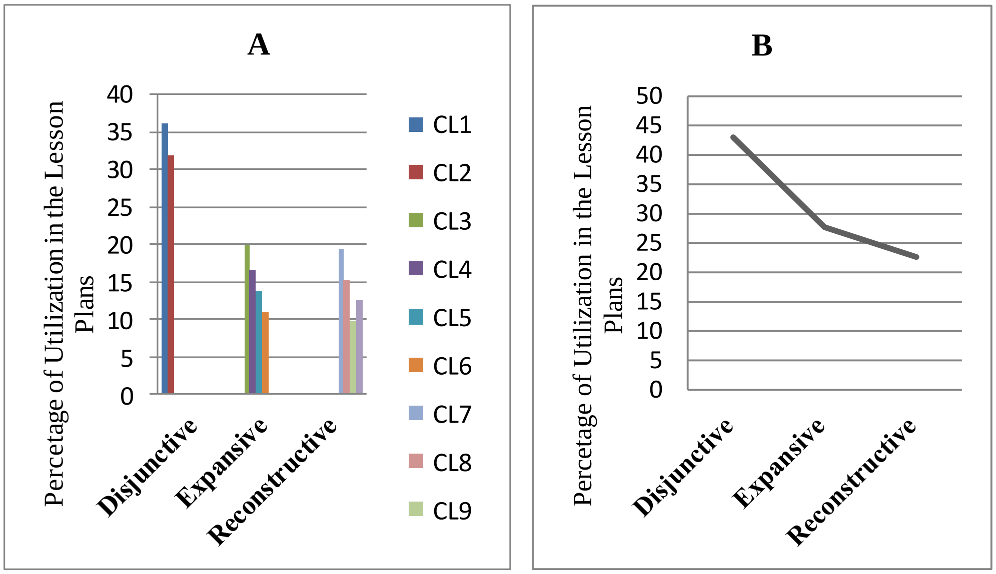

Figure2 A) Rate of using generalization components' items B) Rate of utilization of generalization components

Part A of Figure 2 gives the whole picture of application of items of the generalization strategies in the mathematics instructors' lesson plans. The figure shows thatCL_1 was used more than others while CL_9 was the least. Based on Table 1 and this part of Figure 2, instructors introduced integral by using connected information more than other items of generalization.In addition, they illustrated that they were weak at changing existing schema to apply integral in a wide range.

Moreover Part B of Figure 2 shows that disjunctive generalization was used more as compared to others and reconstructive was the least. This result confirms Tall's opinion about the three levels of generalization such asexpansive, reconstructive and disjunctive. Although constructive is ideal and the highest level generalization, its items such as CL_9 and CL_{10} are used among professional mathematicians (Tall, 2002).

In order to ascertain why there were no suitable utilizations of generalization strategies inside the lesson plans, interview sessions were conducted with twelve instructors. The researchers desired to know if the instructors had logical reasons for applying generalization or vice versa. The results for analysis of interview sessions are presented in the following.

Interview Results

The interview protocol consisted of 12 instructors' perceptions about the definition of generalization, the importance of generalization within the context of integral concept, their examples of generalization, and the grading techniques used when they encountered generalization.

When, the meaning of generalization was asked to the 12 instructors, their answers were generally similar with each other. Most of them believed that generalization is a technique. The excerpts from two instructors are shown below:

> "When we are trying to obtain a general form of equation, we use a technique which named generalization. when we want to address induction, we use generalization as suitable technique" .

Few instructors defined generalization as a rule for working with theories and less specific criteria. Three instructors asserted that:

> "It means that a new rule for solution the sequence or equation through theorems of mathematics".

However none of the instructors mentioned generalization as a process in mathematical thinking as indicated by Tall (2002, 2011) and Mason et *al* (2010). Thus, the data highlight that the instructors did not know the right meaning of generalization to use in the teaching of integral. They thought that generalization is a technique or a brief method such as an induction to solve problems which is related to sequences and equations. They were not informed about the real properties of generalization such as looking for patterns, finding relationship and making connection in different levels of mathematical thinking (Mason et *al*, 2010). In addition,when the instructors were asked to give an example of generalization in integral as the second question of in the interview, their answers were superficial and far from a real example of generalization.

All of the instructors were asked to give examples which were related to integral concept. Four of them did not give any suitable example to the interviewer. For other eight instructors, 3 of them gave "$\int_a^b f(x)dx = A$ *to obtain the areas of function f(x) and x-axis from a to b"*. One of them mentioned:

> "A polygon is a generalization of triangle or quadratic shapes"

Her answer was a general example which did not pay attention to the integral concept. The given example was the same for two other instructors:

"$\forall x \in [a, b]$ if $f(x) \leq g(x)$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$".

Another pair of instructors responded:

"if $F'(x) = f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$".

In fact they mentioned the first basic theorem of calculus and differentiation which is a rule to solve problems. Although this is a general form of integral properties, it did not indicate any trace of generalization process. An instructor showed that:

"if $\int x\,dx = \frac{x^2}{2} + c$, we can conclude that $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$".

Although his example was better than others, he did not mention how the general form was reachable by using only one example. He lost the activities such as specialization and conjecturing which should be done as the basics activities of generalization process. Furthermore, he forgot to mention $n \neq -1$ as an important condition for the example above. Aside from that, it seems that all of the 12 mathematics instructors who were interviewed did not know how generalization process could be done in the teaching concepts. They imagined generalization as general symbols of functions, theorems and others. Then they considered only algebraic aspect of function without giving example in embodied aspect of integral. However Tall (2012) asserted that integral is presented through two aspects; algebraically and geometrically.

When the instructors were interviewed to about the circumstances of using generalization in the teaching of integral, majority of them believed it was useful and important in teaching process. They could not explain more in detail because of their weaknesses in knowing about generalization implication and posture of suitable examples. One of them said that:

"Not only in integral, but also in most of the field in mathematics generalization is useful and necessary".

He did not explain more when the interviewer asked him. Seven of the instructors demonstrated that generalization was used more in the application of theorems to solve problems. For example, one of them said:

"I think usually we use generalization when we want to use the properties of theorems to solve problems".

Based on the belief of that instructor, he used disjunctive generalization. This kind of generalization has negative effects on the learning of concepts (Tall, 2002), because a familiar context is used to solve more special problems and there is no creativity in this kind of problem solving. Another instructor explained the utilization of partial principles in integral by using generalization:

"I think that we can help students to understand difficult concepts of integral through using generalization. The partial principles of integral can be extended to reach a general theorem for helping students to solve difficult problems. It means that students can solve difficult problems by extending these principles".

When she was asked to explain more about what principles are, she told:

"for example, $\int \frac{1}{u}du = ln\,u + c$ is a principle which we can extend it to solve $\int \frac{2x}{x^2}dx$.".

According to the meaning of principle in mathematics, there is no real principle for integral concepts. Principles are used in geometry and mathematical logic. It seems that she wanted to tell that the formulas of calculating the integral of special function can be applied to solve new problems. However, this is not a useful generalization to help students and it is known as disjunctive generalization.

In answering how generalization can be used in the teaching of integral, four instructors mentioned that they usually use it to teach definition of finite integrals and solving problems of multiple integral. To illustrate, one of them expressed:

"I use generalization to show $\int_a^b f(x)dx = lim_{\Delta x \to 0} \sum_{n=1}^{\infty} f(x)\Delta x$, to solve $\int_a^b \int_c^d f(x,y)dxdy$ and to find area, volume and the length of curves".

This response was given without considering the quality of using generalization. Their answers were about the situations which generalization can be used, but the interviewer wanted to know how generalization process can be done from the viewpoint of the instructors. According to the responses, dealing with algebraically aspect of integral was more apparent among instructors than graphically (geometrically). There were weak signs of using expansive generalization in symbolic aspect (Tall, 2002, 2004; Hashemi et *al*, 2013b). Although their efforts were likely to be expansive generalization, there was no remarkable sample of using this strategy among the instructors.

The forth main question asked was about the instructors' experiences of using generalization in the lesson plans of integral. Three of the interviewees told that they did not use generalization in their lesson plans for integral at all. Other nine instructors claimed that they used generalization in writing lesson plans. When those instructors were asked to say when and where they used generalization in their lesson plans, they mentioned to use theorem in integral. For instance:

> "I usually use generalization when I want to solve integral problems through teaching process, because I use formulas and basic theorem of calculus and differentiation in the problem solving activities".

They used the properties of theorems to solve integral problems. This kind of application of generalization called disjunctive is not useful in overcoming learning difficulties the teaching of integral (Hashemi et al, 2013 b). One of them mentioned that:

> "Although I use generalization in the teaching of integral, I often use it in the teaching of analytic geometry and linear algebra when I want to solve problems of three-dimensional by using properties of two-dimensional".

The results of analysing lesson plans showed that although they claim using it in the teaching process, the instructors did not use real generalization strategies in their lesson plans. The inadequacy arose because of their misunderstanding and ignorance of real sense, importance and effectiveness of generalization in the teaching process.

The fifth main question asked if generalization can be considered as a part of lesson plan for teaching integral. Three of them told that it was not necessary to use it in the lesson plans. Their reason reflected generalization was difficult and it was high level for students to deal with. However, they did not express any clear definition and information about high level and difficulties of generalization and its utilization in the teaching of integral or other concepts of calculus.

Most of the interviewees told that generalization can be used in lesson plans. To illustrate, one of them asserted that:

> "When I expressed the theorems and formulas and use them in the solving of problems, I can apply generalization in my teaching process".

Although they demonstrated that using generalization was useful in the lesson plan, they still did not sufficiently use it in the samples lesson plans.

The reasons of not using generalization in the teaching of integral were related to their understanding of generalization. They were not informed about the exact interpretation of generalization. They regarded generalization as a short method instead of logical process. Nobody used specialization and conjecturing as basic elements of generalization process. Majority of them used the generalization which is named as disjunctive which has less impact on students' learning of concepts. Based on the responses to the interview questions, a few instructors tended to deal with symbolic aspect of presenting integral by using expansive generalization. However, they did not display more performance of using this generalization in symbolic aspect.

In general, most of the mathematics instructors at undergraduate level did not use generalization in their lesson plans for integral. First, from the interviews, most of the mathematics instructors did not recognize the genuine functionality of generalization. They regarded generalization as a technique, whereas it is a logical process in mathematical thinking and problem solving. Furthermore,they did not consider the properties of generalization such as making connections and relationships between special cases or examples in order to predict a general pattern. In addition, the instructors did not attempt to make connection between different concepts of calculus which are related to integral by generalizing students' previous knowledge to overcome the learning difficulties of integral.

Second, the instructors used familiar context to solve new problems of integral and they believed it as real generalization. As explain before, they usually use disjunctive generalization which is a low level generalization without any remarkable effect on students' achievement in the learning of integral. Besides, the most percentage of generalization used is disjunctive generalization as shown in Table 4.

Next, there was an intention to use generalization in the symbolic aspect of presenting integral concepts among instructors. According to Tall (2002, 2008), using generalization in one aspect of mathematical world without any changing in existing schema is known as expansive generalization. However, there was no remarkable example of the use of expansive generalization in this study. Only weak traces of expansive generalization can be observed in the data.

Finally, there was no indication of using reconstructive generalization in the interview data. Although it is the best and more effective generalization to increase students' learning of calculus, nobody gave an example which was related to this kind of generalization.

■5.0 CONCLUSIONS

The results revealed that mathematics instructors at undergraduate level usually use disjunctive generalization than other strategies in the teaching of integral. However, this kind of generalization has no remarkable effect in improving students' achievements. Expansive generalization was used less than disjunctive although it is more useful than the latter. It should be noted that reconstructive generalization is an ideal generalization to help students; however, in this study it was rarely used. This is a powerful generalization to overcome students' difficulties of calculus (Tall, 2008; Hashemi *et al*., 2013a). Thus, there was a lack of generalization usage in the teaching process of integral concepts.

Generally, the results show that according to instructors' lesson plans, most of undergraduate university instructors did not pay attention to the role of generalization in their teaching process. The results of interviews indicated that there was a weak understanding of roles and applications of generalization among the instructors. In addition, most of instructors did not know the real meaning and process of generalization to be used in the teaching of integral concept. Findings support the needs of developing generalization as an important property of mathematics by mathematics instructors (Mason, 2010). The generalization strategies have the potential to help students and instructors in the process of teaching and learning of mathematical concepts. Tha lack of sufficien time to see instructors's teaching in their class was main limitation of this research. This research suggests that it will be worth to study the impact of generalization to students' learning for future investigations. In addition, future studies can also investigate the impact of using generalization strategies based on the feedback of teaching activities.

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