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The Development of Computational Thinking and Mathematics Problem Solving Skills Through Mathematics Modelling Activities

Lim Kar Teck, Abdul Halim Abdullah*, Mahani Mokhtar, Syed Abdul Hakim Syed Zainuddin, Sharifah Nurarfah S. Abd Rahman

School of Education, Faculty of Social Sciences and Humanities, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia

*Corresponding author: p-halim@utm.my

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Abstract

The latest TIMSS report highlighted that secondary school students from Malaysia did not perform well in this global assessment on mathematics achievement. Some studies have pointed out that students' mathematics achievement can be improved by equipping them with computational thinking. Therefore, this study examined the development of computational thinking skill and mathematics problem-solving skills via mathematics modelling activities among secondary students in Malaysia. The research instrument used for the assessment of computational thinking was adapted from UK Berbras Challenges and CAS Barefoot Team from United Kingdom. In addition, rubrics for development of mathematics problem solving skill was adapted from Polya problem solving model. This study adopted qualitative approach and the modelling activities were conducted for six weeks. The activities were introduction of modelling, individual modelling task and group modelling task. Data were also collected based on results of computational thinking test and mathematics test. Data from observation and interviews were analyzed, and the scores obtained from both tests were compared and analyzed as well. Based on the findings of the study, it could be concluded that the students progressed at different levels during the development of their computational thinking skill and mathematics problem-solving competency via mathematics modelling activities. These results indicated that mathematics modelling activities can be conducted at schools to improve students' performance in Mathematics.

Keywords: computational thinking, mathematics modelling activities, mathematics problem solving

Abstrak

Laporan TIMSS terkini menunjukkan bahawa murid sekolah menengah rendah di Malaysia tidak menunjukkan prestasi yang baik dalam penilaian global mengenai pencapaian matematik ini. Beberapa kajian telah menunjukkan bahawa pencapaian matematik pelajar boleh dipertingkatkan dengan melengkapkan mereka dengan pemikiran komputasi. Kajian ini mengkaji pembangunan kemahiran berfikir komputasi dan kecekapan penyelesaian masalah matematik melalui aktiviti pemodelan matematik dalam kalangan pelajar menengah di Malaysia. Instrumen kajian yang digunakan untuk penilaian bagi pemikiran komputasi yang telah diadaptasi daripada *UK Berbras Challenges and CAS Barefoot Team* dari United Kingdom. Selain itu, rubrik pembangunan kemahiran menyelesaikan masalah matematik telah diadaptasi daripada model penyelesaian masalah Polya. Kajian ini menggunakan pendekatan kualitatif dan aktiviti pemodelan dijalankan selama enam minggu. Aktiviti-aktiviti tersebut ialah pengenalan pemodelan, tugasan pemodelan individu dan tugasan pemodelan berkumpulan. Data juga dikumpul berdasarkan keputusan ujian pemikiran komputasi dan ujian matematik. Data daripada pemerhatian dan temu bual telah dianalisis, dan markah yang diperoleh daripada kedua-dua ujian telah dibandingkan dan dianalisis juga. Berdasarkan dapatan kajian, dapat disimpulkan bahawa pelajar berkembang pada tahap yang berbeza semasa pembangunan kemahiran berfikir komputasi dan kecekapan menyelesaikan masalah matematik melalui aktiviti pemodelan matematik. Keputusan ini menunjukkan bahawa aktiviti pemodelan matematik boleh dijalankan di sekolah untuk meningkatkan prestasi pelajar dalam Matematik.

Kata kunci: Pemikiran komputasi, aktiviti permodelan matematik, penyelesaian masalah matematik

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■1.0 INTRODUCTION

Programme for International Student Assessment (PISA) (2006) is introduced globally with the aim to produce students who possess high mathematics literacy. Literacy in Mathematics is crucial to ensure development of the country in the aspects of technology and economy. Similarly, the main goal of mathematics curriculum in Malaysia is to discover and develop the learners who could think mathematically. To illustrate, it was explained and defined as the process of consuming mathematics thinking extensively to distinguish the relationships between the variables to resolve the problem (Aydin & Ubuz, 2014). Ultimately, Ministry of Education of Malaysia aims to produce students who can think independently and critically to solve real world problem. Therefore, problem solving is being emphasized a lot in teaching and learning of mathematics in the school in Malaysia. For instance, in terms of teachers training, "Boston Model" was introduced in the training of teachers in order to highlight about thinking process and metacognition (Nagappan, 2001). Numerous approaches have been introduced by the Ministry of Education in the school curriculum namely trial and error, recognizing patterns, using a table, working backwards and rational reasoning. To deal with the challenges of 21st Century, one needs to be equipped with competent skills to perform well in the applications of robotics and computing. As a result, the topics of computational thinking (CT) has been widely discussed in the field of research and educations. Furthermore, computational thinking has frequently been associated in the problem-solving setting (Román-González, PérezGonzález, & Jiménez-Fernéndez, 2017). CT has widely been associated with STEAM (Science, Technology, Engineering, Arts and Mathematics). CT and STEAM has a corresponding relationship among others (Barr & Stephenson, 2011). The studies showed that CT skills could be developed with better understanding when the students were exposed to scientific and mathematics problem in a stimulated modelling environment (Brennan & Resnick, 2012). Thus, mathematics modelling helps learners to comprehend and understand mathematics with the modelling tasks that is relevant to real life issues (Kaur & Dindyal, 2010). In addition, mathematics modelling enables the possibility to transfer realistic content to mathematics to drive for the usage and conceptualisation of applied mathematics (Jablonka, 2007). Ottesen (2001) stated that mathematics modelling could be leveraged as the tool to explore about mathematics and to develop mathematics thinking. Blum (2007) stated that applied mathematics as well as modelling were being associated to illustrate the meaningful relationships between the area of mathematics and the extra-mathematics component. Mathematics modelling was being highlighted in the national education curriculum of Germany since 2003 (Greefrath, 2016). The syllabus requires the students to apply modelling to solve real life problem. In addition, Kaiser and Sriraman (2006) illustrated specific modelling as modelling in the context of education by establishing the mathematics thinking in the context of the real-world setting. Many studies which were conducted in the United States indicated that mathematics modelling ought to be emphasized in the K-12 education to assist the students in gaining experiences to solve real world problem with mathematics thinking (Asempapa, 2018).

Aside from PISA, TIMSS is conducted every four years to improve education system in participating countries and also with the aim to produce competent global citizen. Eighth grade (Form 2) Malaysian students have participated in TIMSS assessment since 1999. According to Figure 1, it was noticeable that Malaysia performed poorly ever since its participation in this large-scale survey from the year 1999. Based on the data from TIMMS, it can be implied that majority of Malaysian students can only utilize fundamental mathematics concepts in various scenarios (Mullis, 2016). On the contrary, it is suggested that Malaysian students failed to interpret data from various graphs. They failed to apply reasoning and generalizations in solving complex problem.

Figure 1. Trends of 8th Grade Malaysian' Mathematics Achievement (1999-2015)

Mishra et al. (2013) claimed that teachers preferred traditional teaching methods which was spoon-feeding approach, where the learners encountered passive learning. Among the factors which contributed towards the preference of traditional teaching method are due to time constraints and lack of competency of the teachers (Nagappan, 2001). Furthermore, the teachers must rush to complete the syllabus to cater the needs of examination-oriented culture learning environment (Burkhardt, 2006). In addition, while solving mathematical problems in schools, students used the same methods to check back their solutions. Some students also faced difficulties in interpretations of the problems (Aydin & Ubuz, 2014). Moreover, the learners tend to immediately go into the calculation part of the problem without much considerations on the planning part (Carreira & Baioa, 2018). Correspondingly, problem solving has widely been discussed in the context of computational thinking (Korkmaz, Çakir, & Özden, 2017). In addition, Buteau, Gadanidis, Lovric and Muller (2017) also agreed that CT activities enable the students to widen their perception on execution of mathematics problem solving. On the contrary, the previous research showed that there many students still were not able to master computational skill (Papadopoulos & Tegos, 2012). This is because most of the teachers are not exposed to the usage and application of computational thinking. Likewise, Sanford and Naidu (2016) argued that teachers must be trained on preparation and instruction of potential computational thinking education. Fields, Lui and Kafai (2019) stated that students learnt and performed better when the teachers displayed own computational thinking processes and errors during the teaching instruction. However, there is no guideline provided by the Ministry of Education (MOE) on the evaluation standard of computational thinking skill. Thus, it is challenging for the educators to assess the levels of computational thinking among students.

School teachers desired to focus more on the standard procedure or conventional approach in delivering the subject of science rather than to guide the students to discover the 'realistic' content of science (Clement, 2000). This suggests similar scenarios in the context of mathematics as well. Students are trained to solve problems in textbooks only. Even though some of the problems in textbooks includes real life scenarios, the problem-solving process require general key steps in doing the calculation. During the teaching and learning processes, learners are not encouraged to go out of their comfort zone by relating all the considered variables in structuring the problem-solving thought process. Nonetheless, the general problems only allow the students to utilise the known variables and implement their problem solving in a safe environment (English & Lesh, 2003). Most learners were trained to solve problems without many considerations on inferencing the genuine setting of the world (Boaler, 2002). Most of the times, the learners intended to ignore the practical considerations on the components of the real-life situations when they were assigned with mathematics problems (Dewolf, 2014).

Unlike Malaysia, Singapore has introduced the concept of mathematics modelling and Singaporean students have performed well in in TIMSS. Several non-routines, open-ended and real-world problem were introduced in the textbook (Asempapa, 2018). One of the most famous approaches being taught in Singapore would be "Modelling". It helps the students to visualize the relationships between elements. In addition, in terms of assessment, the usage of multiple-choice questions was being reduced to encourage the learners to "talk" in mathematics about their thought process and reasoning to work out the problem (Yeap & Kaur, 2008). According to the previous researchers, mathematics modelling activities could contribute to the development of problem-solving skill by constructing a model via generalising the problem setting. In fact, there is a lack of study in Malaysia about the development of computational thinking skill and mathematics problem-solving via mathematics modelling activities. Thus, it is significant to conduct this research to investigate the development of computational thinking skill and mathematics problem-solving competency of secondary students through modelling-based activities.

The objectives of the study were to investigate

i. the development of the computational skill of secondary students in the subject of mathematics based on mathematics modelling-based activities
ii. the development of mathematics problem-solving skill of secondary students based on mathematics modelling-based activities.

■2.0 METHODOLOGY

The researcher decided to adopt qualitative research design to fulfil the two research objectives which were to identify the development of computational thinking (CT) skill and mathematics problem-solving (MPS) skill through modelling-based activities. Seven students with distinct achievements in the subject of mathematics were selected as respondents of the study. The study was conducted at one of the private and international school in Plentong, Johor Bahru. The school was selected due to its availability of technological infrastructure. Figure 2 illustrated the six weeks modelling activities conducted among the seven secondary students.

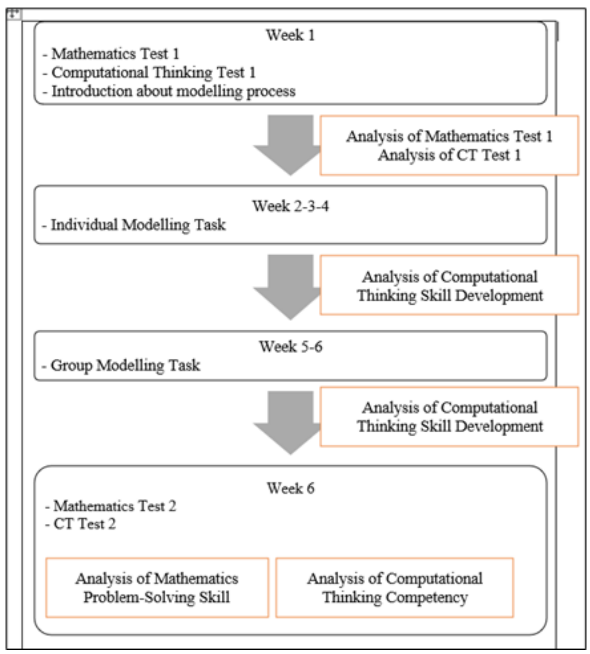

Figure 2. Research Procedure

Both rubrics, which are indicated as Table 1 and Table 2 were designed by the researcher based on the elements of computational thinking model by CAS Barefoot Team and problem-solving model by Polya. The rubrics were verified by lecturers from a university, secondary school mathematics teachers and head of department of the school.

Table 1: Rubrics of Observed Behaviour based on Computational Thinking

Items/Marks	0	1	2
Decomposition	Failed to split the main part into smaller components.	Able to split the main part into smaller components with guidance.	Able to split the main part into smaller components without guidance.
Abstraction	Failed to isolate important and relevant information.	Able to isolate important and relevant information with guidance.	Able to isolate important and relevant information without guidance.
Pattern Recognition	Failed to identify the pattern by revising the work repeatedly.	Able to identify the pattern by revising the work repeatedly with guidance.	Able to identify the pattern by revising the work repeatedly without guidance.
Algorithms	Failed to construct the stacks of blocks of commands for the task.	Able to construct the stacks of blocks of commands for the task with guidance.	Able to construct the stacks of blocks of commands for the task without guidance.
Reasoning	Failed to provide the justification of decisions made based on logical thought process.	Able to provide the justification of decisions made based on logical thought process with guidance.	Able to provide the justification of decisions made based on logical thought process without guidance.
Evaluation	Failed to evaluate the proposed resolution for further improvements	Able to evaluate the proposed resolution for further improvements with guidance	Able to evaluate the proposed resolution for further improvements without guidance

Table 2: Rubrics of Observed Behaviour based on Polya's Problem Solving Model

Items/Marks	0	1	2
Understand the problem	Failed to present or explain about the interpretation of the issue.	Able to present or explain about the interpretation of the issue with guidance	Able to present or explain about the interpretation of the issue without guidance.
Devise a plan	Failed to present or explain about the strategy to cope with the problem.	Able to present or explain about the strategy to cope with the problem with guidance.	Able to present or explain about the strategy to cope with the problem without guidance.
Carry out the plan	Failed to present or explain about the workings steps or algorithms to cope with the problem	Able to present or explain about the workings steps or algorithms to cope with the problem with guidance.	Able to present or explain about the workings steps or algorithms to cope with the problem without guidance.
Looking back the solution	Failed to present or explain about other methods to check the accuracy of the answers.	Able to present or explain about other methods to check the accuracy of the answers with guidance.	Able to present or explain about other methods to check the accuracy of the answers without guidance.

Validity and reliability of the research instruments were the important aspects of a research study. To maintain the validity and reliability of the study, a set of mathematics test was given to six other students with the similar criteria as the chosen respondents before conducting the study. All the subjective questions were designed based on the Polya's problem solving model. Results showed that the six participants agreed that they did not have issues in completing the test in the time given which was sixty minutes. In addition, interview sessions were conducted with the students to cross check about the understanding and the difficulties of the designed questions. Moreover, the mathematics problems and the rubrics of computational thinking and problem-solving model were vetted by ten secondary teachers from various academic and teaching experiences background including the head of mathematics department in the school. Most of the teachers stated that all the items in the rubrics were suitable to be applied in the observation and interviews to be conducted in the modelling-based activities. Therefore, the validity and reliability of the study were emphasized to produce relevant and quality research work.

3.0 DATA ANALYSIS AND FINDINGS

In order to answer the first research objective, data analysis on the development of computational thinking skill were conducted via two mathematics modelling tasks, one for individual task and another one for group task. Seven participants took part in the first modelling task, which was the individual modelling task. Figure 3 shows the individual modelling task which was given to the seven students throughout the three weeks. The learners were required to estimate the prefect angle of basketball free throwing during a National Basketball Association (NBA) game by an NBA player. The modelling framework was adapted from the Common Core State Standards for Mathematics (CCSSM). There were five steps of the modelling process including formulate the problem, model the framework, calculate mathematicsly, draw mathematics conclusion, and validate the resolution. All the students' work were observed and studied according to the different stages of modelling cycle.

Modelling Task (1)

In the NBA basketball game, a player will be awarded a free throw when a foul is committed.

a. Estimate the angle at which a player should throw /shoot the ball to get a perfect free throw (shooting angle) without striking the basketball board.

Figure. 3. Individual Modelling Task

Respondent B was a Year 11 student majoring in science stream. However, he found it difficult to communicate his thought process and his mathematics thinking to others most of the time. According to the transcript of the personal interview, respondent B was able to explain about the stated problem in his own words, which was about finding the prefect angle of a free throw in a National Basketball Association (NBA) game. This implied that he had no issue in interpreting the stated problem. In addition, he also listed out the variables that he considered in the modelling task, namely force applied to the ball, height of the basketball rim, the distance between the player and the rim, mass of the basketball and the height of the player as well. Unlike the usual mathematics problem, the leaners were required to identify the related variables and required measurement that to be applied and associated in the modelling task. This recommended that respondent B was capable in breaking or decomposing the stated problem without the assistance of the facilitator.

By leveraging on the education technology, he discovered the fixed distance between the free throw line and the basketball rim, which was 13.853 meters. Moreover, for the height of the player, he decided to seek for the average height of the NBA players to be used for the part of formulating the framework later. As a result, he intended to exclude the variable of force applied to the ball by only considering the topic of measurement of length and angles. It displayed that respondent B has identified the considered variables and even decided on the selected variables to be studied. It can be concluded that Respondent B was able to do abstraction by eliminating some of the considered variables with no guidance from the facilitator. This sum up the first part of the modelling cycle, which is formulate the problem. Figure 4 illustrated the thought process of respondent B in formulating the problem based on the observation of his works. The height of the basketball rim, the average height of the NBA basketball player, the distance between the free throw line and the rim were included in the Figure 4.

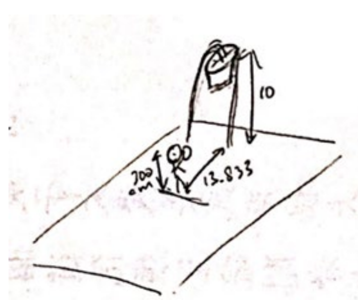

Figure. 4. Formulate the Problem (Respondent B)

Likewise, on the following process, which was the part of modelling the framework. Respondent B decided to apply the graphical method to seek for the resolution. To clarify, he claimed that the trajectory of the ball throwing shall be a curve as illustrated in Figure 5. He demanded that a bell-shaped curve to be considered with a quadratic equation in which the horizontal distance between the turning point and rim was one third of the total horizontal distances between the rim and the player. As a result, the x-intercept and y-intercept of the curve were acknowledged which was 13.853 and 3.44 respectively. In this case, the quadratic equation of the curve could be formed by considering the three coordinates identified including the turning point. This indicated that respondent B intended to do abstraction by considering the selected variables in a structured framework.

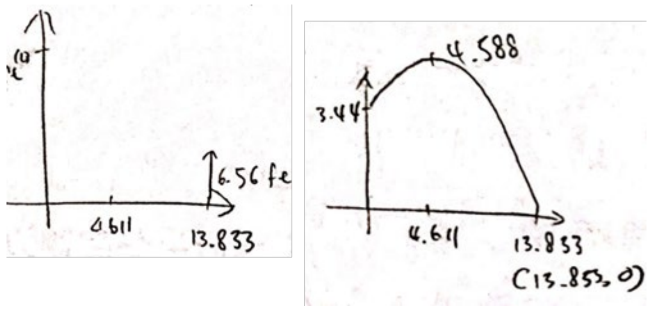

Figure. 5. Model the Framework (Respondent B)

It follows that the mathematics calculation could be done by referring to the mathematics framework that was proposed by respondent B. The quadratic equation was figured out by substituting both coordinates into the quadratic equation. The concept of differentiation was applied by knowing the gradient of the turning point to be zero. Thus, the maximum height of the turning point was identified which was 13.777 as stated in Figure 6. However, the prefect angle of free throw of basketball was not concluded yet.

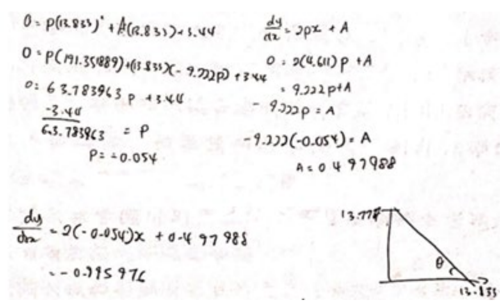

Figure. 6. Tabulation of Conclusion (Respondent B)

In the part of drawing mathematics conclusion, respondent B determined to use trigonometry to obtain the resolution of the problem. According to Figure 7, the shooting angle shall be 44.88º. In this process, the student was competent enough in executing the process of algorithm by involving calculation on the concept of differentiation and trigonometry by leveraging on his previous learning experience.

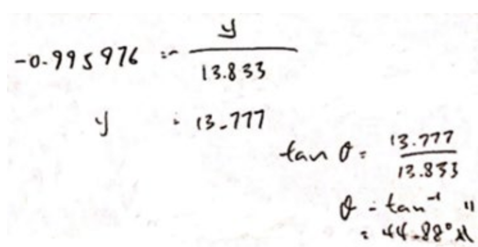

Figure. 7. Resolution (Respondent B)

On the last step of modelling cycle, which was validating the resolution, the interview data showed that respondent B argued that his resolution was quite logical due to the consideration of the trajectory of the free throw instead of considering a straight line based on trigonometry only. Nonetheless, he concluded that shooting angle shall be greater due to other variables that he did not include namely the mass of the ball, the speed of the free throwing and the spin force of the ball. It can be observed that respondent B could communicate mathematically with more encouragement and question probing from the facilitator. Surprisingly, as compared to others, respondent B was the only one that considered another case scenario of jumping free throw which modified and affected the following modelling processes. This showed that respondent B had evaluate his resolution and intended to improve his resolution by validating his mathematics conclusion. Table 3 explained about the level of computational thinking of respondent B based on the interview and observation on works during individual modelling task. All the evaluations were conducted based on the rubrics adapted from CAT Barefoot's instrument including the decomposition, pattern recognition, abstraction, algorithm, reasoning, and evaluation.

Table 3: The Level of Computational Thinking of Respondent B based on Interview and Observation on Works during Individual Modelling Task

Elements	Interview	Observation
Decomposition	2	2
Abstraction	2	2
Pattern Recognition	2	2
Algorithm	2	2
Reasoning	1	1
Evaluation	2	1

Respondent G was the least performing student among the seven participants. Her usual obtained marks for both tests of mathematics was about 50-70 marks. In the interview, she was required to describe the stated problem in her own words. She did not sound confident when she spoke and shared her understanding of the problem. Yet, despite the nervousness, she was competent enough to deliver her descriptions of the problem which has no issues in interpreting it. In her works, she written down a list of variables to be considered as illustrated in Figure 8. Respondent G could decompose the problem into several parts that to be manageable with the listed variables to be studied without the assistance of the facilitator.

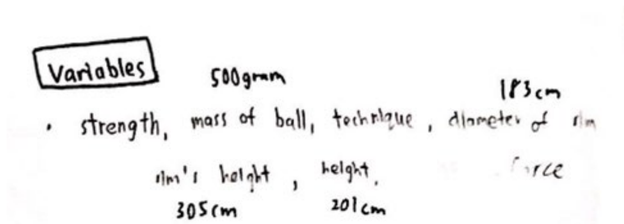

Figure. 8. Consideration of Realistic Variables (Respondent G)

Yet not all of the factors were chosen for tabulation later. The considered factors were the height of the rim and the free throw distance. As compared to respondent B, she did not include the height of the player in the mathematics calculation. The way she modelled the framework would be as illustrated in Figure 9. It was a right-angled triangle which the vertical distance of the basketball rim and the horizontal distance between the free throw line and the rim were included in the stated Figure. She argued that only the main factors to be contained which she filtered out some other considerations during the process of modelling framework. It was followed by the process of producing the mathematics output based on the mathematics concepts used which were trigonometry by assuming that the path travelled by ball of free throwing to be a straight line. Respondent G was skillful to involve the main factors into the part of algorithm. The final answer for the prefect angle based on her calculation would be 32.29º.

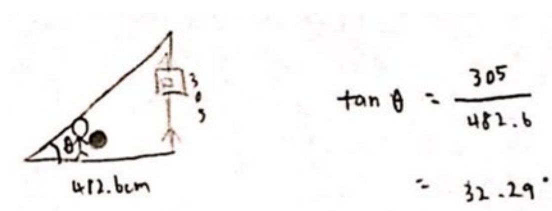

Figure. 9. Model the Framework (Respondent G)

Moreover, in the interview, when she was asked about her own evaluation on her mathematics conclusion, she mentioned about: "Not that logical. It is way too small as compared to others.". That was how she replied in the interview. She seems to have difficulties in defending her own mathematics resolution when she was required to elaborate more on her logical thought process during the modelling cycle. Further, she claimed that she had not ideas on what to improve and enhance on her works if she was going to repeat the whole modelling process again for the same task. It could be observed that her reasoning and evaluation skills based on CT was not strong enough to be practiced and applied to the best of her ability. Table 4 showed the level of CT skills of respondent G based on the observation on her individual modelling task and interview conducted after the task.

Table 4: The Level of Computational Thinking of Respondent B based on Interview and Observation on Works during Individual Modelling Task

Elements	Interview	Observation
Decomposition	2	2
Abstraction	2	2
Pattern Recognition	0	0
Algorithm	0	0
Reasoning	0	0
Evaluation	0	0

Figure 10 shows the group modelling task which was finding the number of passengers stuck in the half of the total distance of the traffic jam which was also the middle part of the longest highway at Johor. There were four participants in group 2, which was respondent B, D, F and G. Unlike the previous individual modelling task, respondent G was the first one to lead the discussion on breaking down on the information to search based on their interpretation and understanding on the given problem. In addition, with confidence she mentioned during the interview: "The question required us that to find longest highway in Johor. If there is a traffic jam, about half of the total distance of the highway…. all the cars are stuck in the jam… standstill… we need to estimate number of people in the cars in the highway.". This statement was flawless in elaborating the description of the problem. In the interview, respondent B mentioned that only the average length of car was placed into mathematics calculations though other type of vehicles namely bus and lorry had been considered at the first place.

Modelling Task (2)

During a long weekend holiday, in the longest express highway at Johor, there were a traffic congestion at the middle of the highway for about half of the total distance of the highway. All the cars were stuck at the traffic jam and it was completely standstill.

a. Estimate the number of people in the car along the traffic jam.

Figure. 10. Group Modelling Task

Similarly, respondent G argued that weather condition during the traffic jam was mentioned in the discussion but no follow up taken on the stated variable due to the reason of lacking required knowledge and skills to model the framework. After discussion, they listed out a list of variables to be discussed including the average number of people in the car, the total distance of the highway, the duration of the highway and the speed of the car as indicated at Figure 11. This implied that they went through the process of abstraction by deciding on the relevant variables to be modelled.

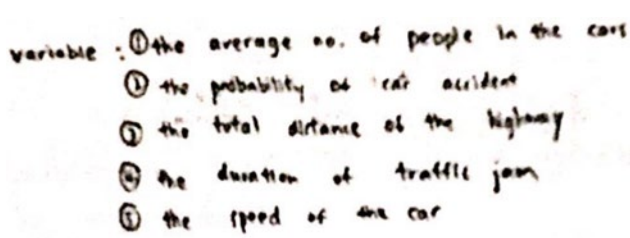

Figure. 11. Consideration of Realistic Variables

This group assumed that Senai-Desaru Expressway (SDE) to be the longest expressway in Johor. Thus, the total distance of SDE was taken into consideration which was about 77 km. There were three lanes along the highway. As a result, the total distance of the traffic jam should be 115 500 m after considering the half value of the total distance of expressway with three lanes. Moreover, the average value of length of vehicle was taken as 4.6 m including the gap between the cars. After tabulation, there were total of 25168 cars. Consequently, averagely, there were three persons in a car by contemplating the possibility of the commonly used cars in Malaysia. Ultimately, there were 75324 passengers stuck in the traffic jam at the longest express way at Johor as indicated at Figure 12.

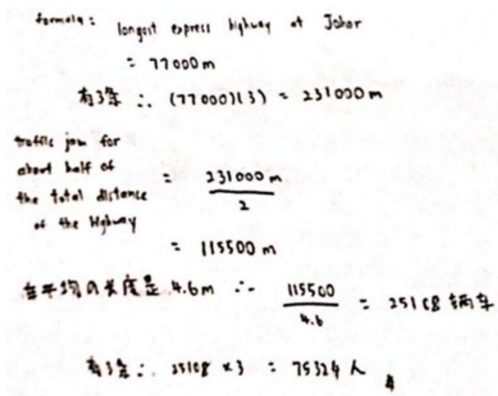

Figure. 12. Tabulation of Conclusion

All of them agreed during the interview, that the resolution was quite logical with all the realistic contents consideration. However, respondent G argued that the longest expressway at Johor might not be SDE though all the mathematics discussion that was carried out was supported by strong mathematics reasoning. Similarly, respondent F stated the longest expressway should be North-South Highway after thorough reflection. This suggested that there was an improvement on the development of the reasoning and evaluation part of CT skills for both respondent G and F since both could provide strong justifications with some practical works done and solid outputs backed by mathematics concepts and realistic contents consideration. Table 5 indicated the development of CT skills of respondent B and G based on interview and observation on works during both modelling task. It could be observed that respondents B & G showed progress in developing CT skills through modelling-based activities based on the analysis on the observation on works and interviews conducted.

Table 5: The Development of Computational Thinking of Respondent B and G based on Interview and Observation on Works during Individual and Group Modelling Task

	Elements	Individual Modelling	Group Modelling	Interview 1	Interview 2
Respondent B	Decomposition	2	2	2	2
	Abstraction	2	2	2	2
	Pattern Recognition	2	2	2	2
	Algorithm	2	2	2	2
	Reasoning	1	2	1	2
	Evaluation	1	2	2	2
Respondent G	Decomposition	2	2	2	2
	Abstraction	2	2	2	2
	Pattern Recognition	0	2	0	2
	Algorithm	0	2	0	2
	Reasoning	0	2	0	2
	Evaluation	0	1	0	1

Two CT tests were conducted before and after the treatment of both modelling tasks. Both CT tests were created and adapted based on UK Bebras Challenges which was an international competition around 30 countries and it emphasised on CT skills to be leveraged on solving problems. In this study, since the all the participants were year 11 students with the average of 17 years old, thus the past year questions for the group for elite (age 16-18) were chosen to be adapted as the instrument of the CT tests. There were total of six questions including three most difficult questions, two not so difficult questions and one easy question for both CT tests. The results of the CT tests were illustrated in Table 6.

Table 6: Comparison of Results of all Respondents in both CT Tests

Respondents	CT Test 1	CT Test 2	Difference
A	10	21	11
B	6	44	38
C	17	44	27
B	6	32	26
E	17	21	4
F	-4	21	25
G	-10	44	55

According to Table 6, it could be concluded that all seven respondents were able to develop and build up their CT skills with different pace and progress through taking part of mathematics modelling activities that lasted for six weeks. This statement can be supported by the previous academic research works. Brennan and Resnick (2012) argued that the leaners can develop the computational thinking skill when they were exposed to some scientific and mathematics problem in a modelling platform. Similarly, Basu, Biswas and Kinnebrew (2017) also claimed that students learnt better about computational thinking by participating in the modelling-based activities. Consequently, respondent B and C were the top scorers in the subject of additional mathematics and mathematics. This suggested that the learners with higher achievement in mathematics able to master better computational thinking skill when compared to students with lower academic performance in mathematics. Based on previous research studies, Gadanidis (2017) claimed that there was a relationship between computational thinking competency and mathematics thinking. Likewsie, Weintrop et al. (2016) stated that there were a close association between mathematics ability and computational thinking skill. Similarly, Saritepeci and Durak (2017) also argued that students with higher achievements in mathematics were able to develop and build up higher level of computational thinking skill.

To answer the second research objective, the observation was conducted on how the students solved the mathematics problem for data analysis on the development of MPS. Figure 13 indicated the mathematics problem handled by respondent C during Test 1. This problem was about the topic of probability distribution. The question stated that a test paper consists of 40 questions. Each question is followed by four choices of answer, where only one of these is correct. The first part of the problem illustrated that Salma answers all the questions by randomly choosing an answer for each question. The leaner was required to estimate the number of questions she answered correctly. According to Figure 13, she wrote down the equation of $^{4}C_1 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^3$. This implied that respondent C knew it well that this problem was associated with the topic of probability distribution in which the formulae of binomial distribution ought to be conducted for calculation. However, the moment she wrote $^{4}C_1$ in the equation led her to a wrong direction in solving this problem. This question was indeed about the topic of probability distribution as she recognised it. Conversely, she did not fully understand the problem as the ultimate goal was to find the number of questions to be answered correctly. Leveraging on the formulae of binomial distribution could only head to the output of probability or chances. On the contrary, in the equation that she proposed, the appropriate value to be applied would be the probability of answering one correct answer which was $\frac{1}{4}$ and it was regarded as the p in the formulae. This finding suggested that she understood the problem on a certain part only. As for the second part of the problem, it was about finding the probability of answering 36 questions correctly when one answers 30 questions and randomly choose an answer for each of the remaining ten questions. According to Figure 13, it could be observed that she listed out the following equation, which was $^{10}C_6 \times \left(\frac{1}{4}\right)^6 \times \left(\frac{3}{4}\right)^4$.

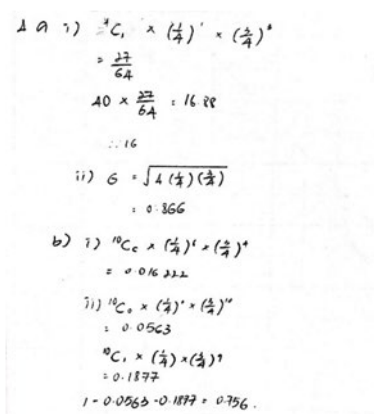

Figure.13. Working Steps of a Probability Distribution Problem (Respondent C)

Unlike the first part of the question, she interpreted the other part of problem accurately. It stated that Salma already answered 30 out of 40 questions. In order to get the distinction in the test, she needs to answer 90% of the questions correctly, which was 36 questions. The value of six to be tabulated by subtracting 36 with 30. The thought process behind the number would be the probability of answering 30 questions had been fixed. In another words, six more questions could be considered out of the remaining ten questions for tabulation of probability of success rate. This showed that she has no issues of handling the first three part of the MPS's process which was understand the problem, create a strategy, and conduct the plan. In the interview, she was asked about the validity of the resolution. She replied in the following statement "I checked my workings by converting the fraction into decimals…. One over four the fraction… I change to zero point two five… and another one to zero point seven five…and I get the same answer". Overall, to analyse the level of MPS skill of respondent C, it could be concluded that she has a better level of MPS as compared to other respondents. Figure 14 indicated the problem with the topic of probability distribution delivered in Test 2 by respondent C. The problem was designed with the topic of probability distribution. This was to make comparison more possible by comparing the problem with the same topic and marks allocation but different context in both mathematic tests. The stated problem described about the probability of owning a national car among the families in a district area. Based on the problem, it was given that three out of five families own a national car. Further, if ten families were chosen at random from the district, the value of the probability that eight families own a national car ought to be investigated.

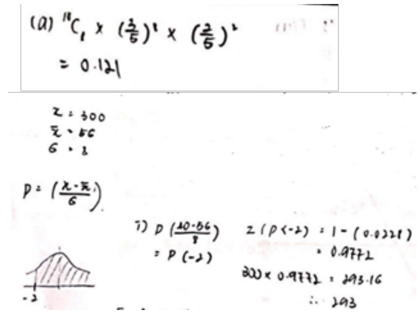

Figure. 14. Working Steps of a Probability Distribution Problem (Respondent C)

As compared to her problem-solving process in the similar question in Test 1, she showed improvement in interpreting the descriptions of the problem. This could be further elaborated by referring to her working steps on Figure 14. She successfully figured out the concepts to be applied in the scenario, which was the formulae of binomial distribution. Moreover, the success rate and failure rate of choosing a family to own a national car were identified and associated accurately which were 3/5 and 2/5. It was followed by her part of mathematics tabulation. As a result, she obtained the right resolution for the indicated problem. The following part of the problem was about number of students passing the test if the passing mark was 40 and there were total of 300 students sat for the test. Further, the marks obtained follow a normal distribution with a mean of 56 and a standard deviation of eight. Respondent C started off by using mathematics notions and symbols to signify the information that were given. For instance, the symbol of normal value, mean and standard deviation were presented on the paper based on her understanding on the topic. In addition, she successfully converted the normal value to a standard score of negative two. Moreover, she also sketched out the normal distribution graph to visualise the area of the graph which was also regarded as the probability of the event that to be found. Consequently, the number of students that passed the test was found by listing out the working steps perfectly. This implied that she was executing the appropriate strategy by using the right mathematics concepts and theories in handling the problem. In the interview, she mentioned that "After I got the answer…I use the backward workings to start off with the number of students passed the test…. until I got back the passing mark… which was given at the first place…. To check my answer". This showed that she could check back her workings steps independently to validate her resolution. Table 7 illustrated the development of her mathematics problem solving skills in test 1 and test 2.

Table 7: The Development of Mathematics Problem-Solving Skill of Respondent C based on Interview and Observation on Works during Mathematics Test 1 and Test 2

	Elements of Polya's MPS Model	Test 1	Test 2	Interview 1	Interview 2
Respondent C	Understand the Problem	2	2	2	2
	Devise a Plan	1	2	1	2
	Carry Out the Plan	1	2	1	2
	Looking Back the Solution	0	1	1	2

Overall, according to the analysis of the development of MPS skill based on the observation on students' work, interviews, and mathematics test scores, it can be concluded that all respondents showed considerable progress in improving MPS skill via mathematics modelling activities. Respondent C showed minor improvement in building up MPS skills based on the stated analysis method. In the context of MPS, most of the respondents performed well on the part of interpreting the descriptions of the problem as most of them scored two marks according to the rubrics indicated in Table 3. This implied that most of them could understood fully about the context of the mathematics problem independently. In addition, most of them failed to express alternative methods to check the accuracy of the answers based on the analysis of the observation on works and interviews. This resulted them to obtain zero or one mark in the part of looking back the resolution in the MPS model. Table 8 illustrated the comparison of results of all respondents in both mathematics tests.

Table 8: Comparison of Results of all Respondents in both Mathematics Tests

Respondents	Maths Test 1	Maths Test 2	Difference
A	44	66	22
B	42	68	14
C	64	82	18
B	35	46	11
E	40	70	30
F	33	72	39
G	24	72	48

Based on the analysis of observation of students' work and test score of mathematics tests, it can be concluded that all respondents developed their mathematics problem-solving skill through mathematics-based activities. This notion can be supported by the previous research studies. Mathematics modelling task trained the leaners to comprehend the application of mathematics by leveraging on the mathematics concepts and theories in the real-life context (Janlonka, 2007). Furthermore, when the students were exposed about the mathematics modelling experiences, they ought to be more adapted into the understanding of the concepts of STEM subjects (Banks & Barlex, 2014). Likewise, Ottesen (2001) indicated that there were a close relationship mathematics modelling and mathematics thinking.

In the context of Polya's problem solving model, results showed that all respondents indicated low achievements in the part of looking back the solution. Most of them only obtained zero or one mark based on the rubric illustrated at Table 2. It suggested that most of the respondents either failed to present alternative resolution to validate the answer or they required the guidance from the facilitator to produce logical interpretation in validating their resolution. This implied that the learners were not trained to communicate with mathematics reasoning in the usual classroom instruction. Based on the previous research studies, Carreira and Baioa (2018) indicated that the leaners tend to apply the same strategies in checking back the accuracy of their answers. There were possible reasons in resulting the learners being discouraged in communicating their mathematics thinking. For instance, Burkhardt (2006) stated that teacher centered learning and teaching was conducted at most of the classroom which resulted in inactive learning. Further, Nagappan (2001) argued that the students were trained to master mathematics problem-solving skill in the examination-oriented culture learning platform. This caused the students to be taught in jumping straight into the calculations part when they tend to neglect the importance of validating their answer with alternative methods.

In terms of the development of mathematics achievements, respondent G was viewed as the students that progressed the most in the comparison of the results based on the rubrics on the analysis of observation and interview. Among the seven participants, respondent G was the students who usually gain the lowest marks among them which was ranged around 50-65 marks in the topical tests. After going through six weeks of mathematics modelling activities, she showed improvement in handling mathematics problem in Test 2. This implied that the mathematics achievements of the students could be enhanced when they participated in the modelling tasks with certain amount of duration (Boaler, 2001).

■4.0 IMPLICATION OF THE STUDY

The findings of this study owned its implications which were beneficial for the educators and educational policy makers in terms of teaching and learning of mathematics. This study indirectly explored the potential use of mathematics modelling activities in the classroom instruction of mathematics to develop computational thinking and mathematics problem-solving competency. Moreover, the findings of the study can be regarded as the teaching and learning resources for the educators in implementing individual and group modelling based activities. As a result, with more teaching and learning resources on the topic of mathematics modelling, teachers have the potential to instill their capabilities in conducting mathematics modelling activities. Furthermore, the instrument of this study can be applied as the guidance for the development of assessment tool for evaluation of computational thinking skills and mathematics problem-solving competency.

In addition, students' participation in mathematics modelling activities in classrooms can help to enhance their understanding of mathematics concepts and theories by communicating the mathematics reasoning. Likewise, Gainsbug (2008) claimed that students would be more proactive in learning mathematics when they could understand better about mathematics concepts via modelling activities. Furthermore, when the learners were exposed to mathematics modelling process, they would be familiar in handling the problem with realistic content. This resulted them to consider realistic variables in resolving the issue. It was vital to associate real world issues to the teaching and learning of mathematics to handle 21st century problem (Smith & Morgan, 2016). As for education policy makers in Malaysia, the findings of this study provided an overview of the importance of mathematics modelling activities to be implemented as the classroom instruction in Malaysian schools. Moreover, the study about the development of computational thinking skill and mathematics problem-solving competency via mathematics modelling activities in the context of the national education syllabuses of Malaysia is rarely to be found in the research field. As a result, the findings of the study can be referred as the guidance in incorporating the element of mathematics modelling into the mathematics syllabus. in addition, to equip trainee teachers and teachers with the appropriate skills set of modelling, workshops can be designed based on the output of this study. This notion can assist in enhancing the competency of Mathematics teachers in Malaysia.

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